

# Some recent results on iterative roots

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## Abstract

An *iterative root* of order  $n \geq 2$  of a self-map  $f$  on a nonempty set is a self-map  $g$  on the set such that  $g^n = f$ . In this talk we discuss some of our recent results concerning the nonexistence of iterative roots—both for arbitrary functions on sets and for continuous functions on topological spaces. A key observation is that if a function  $f$  possesses a special point  $x_0$  (not a fixed point of  $f$ ) such that the second preimage  $f^{-2}(x_0)$  is significantly “larger” than  $f^{-1}(x)$  for all  $x \neq x_0$ , then  $f$  admits no iterative roots. We also explore some implications of this result, including certain density phenomena associated with iterative roots, and its extension to multifunctions. On the other hand, we investigate a fundamental class of multifunctions known as pullback multifunctions, and examine how the iterative root problem for these multifunctions relates to the corresponding problem for single-valued maps. The talk is based on our following recent works:

- [1] B. V. R. Bhat and C. Gopalakrishna, Iterative square roots of functions, *Ergodic Theory Dynam. Systems*, 43 (2023), 2201–2227.
- [2] B. V. R. Bhat and C. Gopalakrishna, The non-iterates are dense in the space of continuous self-maps, *Nonlinearity*, 36 (2023), 3419–3430.
- [3] B. V. R. Bhat and C. Gopalakrishna, Iterative roots of multifunctions, *Fundamenta Mathematicae*, 265 (2024), 141–163.